

Problem 1. Is it correct that if $3 < a < b$ then $a^b > b^a$?

Problem 2. Solve the differential equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(x, t = 0) = 0$, $\frac{\partial u}{\partial t}(x, t = 0) = \sin x$ in the domain $(x, t) \in \mathbb{R} \times \mathbb{R}_+$.

Problem 3. Give an example of 2×2 matrices A, B such that for all $t \in \mathbb{R}$ the matrix $A + tB$ has eigenvalues $\lambda_{1,2}(t) = \pm\sqrt{t}$.

Problem 4. A worker carries two bags. Each of the bags initially contains N nails. Whenever the worker needs a nail, he takes it from a bag picked at random. At some point the worker picks an empty bag. Find the probability that the other bag contains exactly m nails.

Problem 5. Is it possible to paint the edges of an n -gonal prism with 3 colors so that each face has all 3 colors and all the edges of each vertex have different colors if a) $n = 2018$; b) $n = 2019$?