ONLINE ENTRANCE TEST ON ENERGY SYSTEMS MSC PROGRAM: SAMPLE

BLOCK 1. MATHEMATICS

Problem 1.1. Find the derivative with respect to x of the function $f(x) = (7+x)^{(7+x)^2}$

Problem 1.2. For the equation $2xy' + y = 3x$ determine $y(2)$ if $y(1) = 1$

Problem 1.3. Let

$$A = \begin{bmatrix} 1\frac{1}{3} & 0 & -\frac{1}{3} \\ \frac{1}{2} & 3 & -\frac{1}{2} \\ -\frac{2}{3} & 0 & 1\frac{2}{3} \end{bmatrix}$$

Find the maximal eigenvalue of the matrix A^4.

Problem 1.4. Calculate $(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^{2019}$

Problem 1.5. A farmer has 1200 m of fencing and wants to fence off a rectangular field that borders a straight river. He does not require a fence along the river. What are the dimensions of the field that has the largest area?

Problem 1.6. The cylindrical tank with a diameter $2R = 1.8$ m and height $H = 2.45$ m is full of water. How much time is needed for the water to flow out through the hole in the bottom with a diameter $2r = 5$ cm. Cylinder axis is vertical.

BLOCK 2. OPTIMIZATION, BASIC ECONOMY, STATISTICS

Problem 2.1. A product is sold to the price $50 USD, then revenue function is

a) $\mathrm{R}(50) = 50 + \mathrm{x}$;
b) $\mathrm{R}(\mathrm{x}) = 50\mathrm{x}$;
c) $\mathrm{R}(50) = \mathrm{x}$;
d) $\mathrm{R}(\mathrm{x}) = 50 + \mathrm{x}$.

Problem 2.2. The mean of a sample is

a) always equal to the mean of the population;
b) always smaller than the mean of the population;
c) computed by summing the data values and dividing the sum by (n - 1);
d) computed by summing all the data values and dividing the sum by the number of items.

Problem 2.3. Solution set for a group of linear inequalities is classified as

a) concave set;
b) convex set;
c) loss set;
d) profit set.

Problem 2.4. Number of ordered pair values (x,y) to satisfy linear equation ax + by = c are

a) finite;
b) infinite;
c) zero;

d) rational expression.

Problem 2.5. 5. If there is a probability of 5% in how many cases would a result arise solely due to chance?

a) 5/100;
b) 95/100;
c) 50/50;
d) None of these.

Problem 2.6. Which one of the following is an example of a positive externality?

a) a government budget surplus;
b) a motorway improvement scheme;
c) more exports than imports;
d) noise pollution.

BLOCK 3. THERMODYNAMICS

Problem 3.1. Provide concise answers to the following questions. Calculation developments if any, should be kept to a minimum showing essential steps, and the final result highlighted.

1. what are the main modes of heat transfer?
2. what is a state function in thermodynamics?
3. let w the differential form: $w = (y^3 - 6xy^2)\mathrm{d}x + (3xy^2 - 6x^2y)\mathrm{d}y$; is w an exact differential form on $\mathbb{R}^2$? Calculate its integral on the arc circle delimited by the points A in (1, 2) and B in (3, 4).
4. draw the (P, T) phase diagram of water, indicating all phases, and particular points.

Problem 3.2.

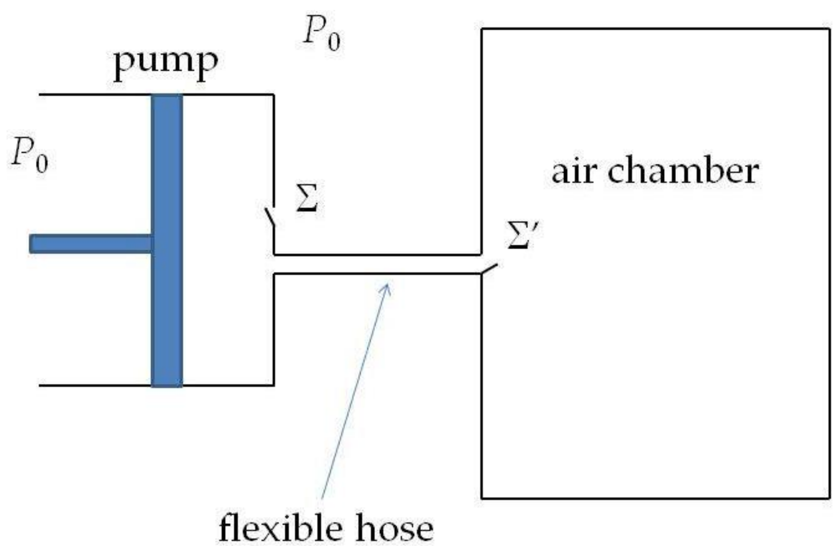

Figure 1: Air chamber connected to a pump.

Consider an air chamber of constant volume $V_0 = 9\ \ell$, initially containing air at atmospheric pressure P_0 = 1 atm = 10^5 Pa, and temperature $T_0 = 300$ K, and receiving more air at constant temperature T_0 through a pump until the pressure reaches P_1 = 2 atm. The pump is connected to the chamber through a flexible hose with a volume assumed to be negligible compared to that of the chamber; Σ and Σ' are two valves, and Σ' opens only when the pressure in the pump reaches the level of the pressure within the air chamber.

Each pump stroke is characterized by 3 phases:

- *admission* of a volume V_a of air at temperature T_0 and pressure P_0 (Σ opened and Σ' closed);
- *adiabatic compression* (Σ closed and Σ' closed) until the opening of Σ';
- *injection* of compressed air into the air chamber

Assumptions: i/ quasi-static changes so that there is no friction, hence no energy loss; ii/ thermal equilibrium between the air inside the chamber and the outside environment (at temperature T_0); iii/ ideal gas model for the air with $\gamma = 7/5$.

1. Calculate:

a) the number Δn of moles of air injected in the air chamber;
b) the pressure variation δP in the air chamber;
between two *successive* strokes.

2. Determine the pressure P_k in the chamber after the k^{th} stroke, as a function of P_0, V_0, V_a.

Problem 3.3. The Maxwell-Boltzmann distribution characterizes the molecular speed distribution in ideal gases; it reads:

$$f(v)\mathrm{d}v = Cv^2 \exp\left(-\frac{mv^2}{2k_BT}\right)\mathrm{d}v,$$

where C is the normalization constant.

- briefly name and define the various physical terms in the formula above;
- calculate the normalization constant C;
- calculate $\langle v \rangle \times \langle 1/v \rangle$, where $\langle \cdot \rangle$ denotes the statistical mean.

Problem 3.4. Statement of the first law of thermodynamics.

Problem 3.5. Thermodynamic definition of entropy.

Problem 3.6. Consider a gas contained in a sealed box of volume 1 L at pressure 1 bar; its temperature is T = 27°C. Assuming the ideal gas law, find the number of moles for this gas.

BLOCK 4. ELECTRIC MAGNETISM

Problem 4.1. The Wheatstone bridge is a useful circuit for resistance-based measurements. Figure 2 shows the basic circuit. The bridge is balanced when the voltage between A and B is zero. Which is the value of $\mathrm{R_x}$ that balances the bridge, if the other resistance values are $\mathrm{R_1} = 5$ Ohm, $\mathrm{R_2} = 10$ Ohm, $\mathrm{R_3} = 20$ Ohm?

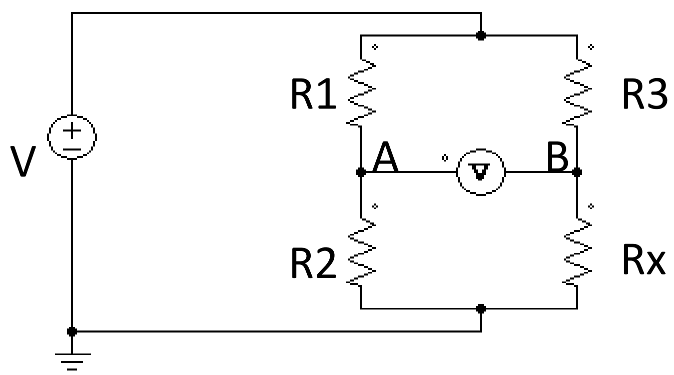

Figure 2.

Problem 4.2. The gate of POWER MOSFET or IGBT can be modelled as a capacitor. *Figure 3* shows a typical input circuit of a POWER MOSFET. The driver, which can be considered as a voltage source with an internal resistance, delivers an amount of charge during the turn-on and turn-off processes. In a normal cycle, the POWER MOSFET is on during a time interval and then is turned off waiting for the next cycle. *Figure 2* shows the waveforms of the voltage source that produces this behavior.

Which the average power losses in the internal resistance if $R \cdot C \ll T$?

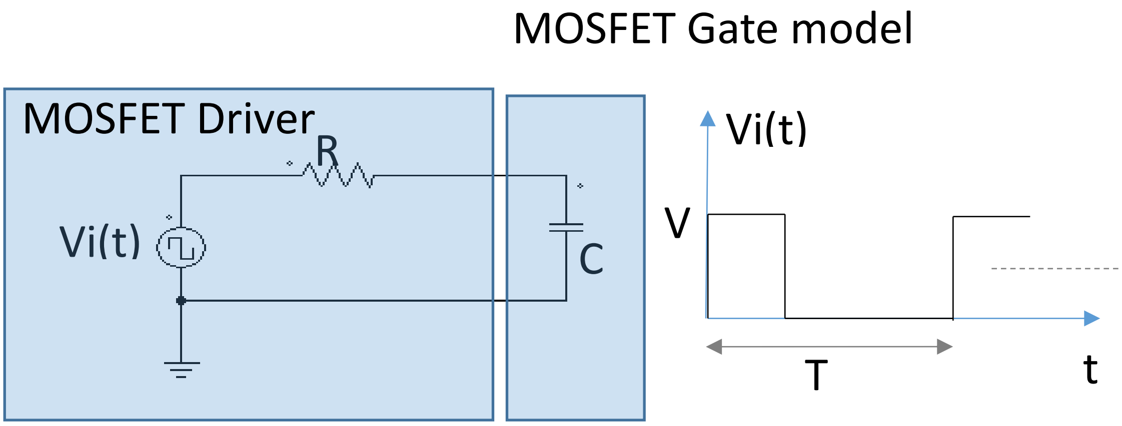

Figure 3.

Problem 4.3. *Figure 4* shows a low-frequency DC transmission system that includes the generator internal resistance (R_i), the losses in the transmission line (R_t) and the load (RL). Which is the value of RL for the maximum energy transfer from the source to the transmission line?

Problem 4.4. And which is the efficiency for this condition (*Figure 4*)?

Problem 4.5. If $R_i + R_t = R_g$, and the power transfer efficiency is 0.99 (defined as the ratio between the power at RL and the power delivered by V_i), which is the value of RL?

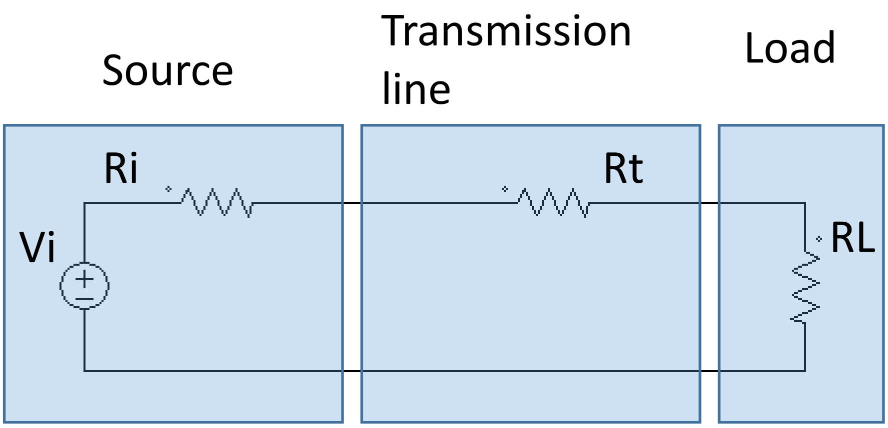

Figure 4.

Problem 4.6. A supercapacitor is charged with a constant current-constant voltage method (CC-CV). The supercapacitor is initially discharged ($V_{sc} = 0$) and it is charged at a constant current, $I_{ch} = 10$ A. When the voltage in the supercapacitor terminal (V_{sc}) reaches $V_{lim} = 2.6$ V, the supercapacitor is charged at a constant voltage $V_{ch} = 2.7$ V. *Figure 5* shows the charging system and the capacitor model, which is represented by a capacitance $C = 100$ F and a resistance $R_i = 0.01$ Ohm.

When the supercapacitor can be considered charged?

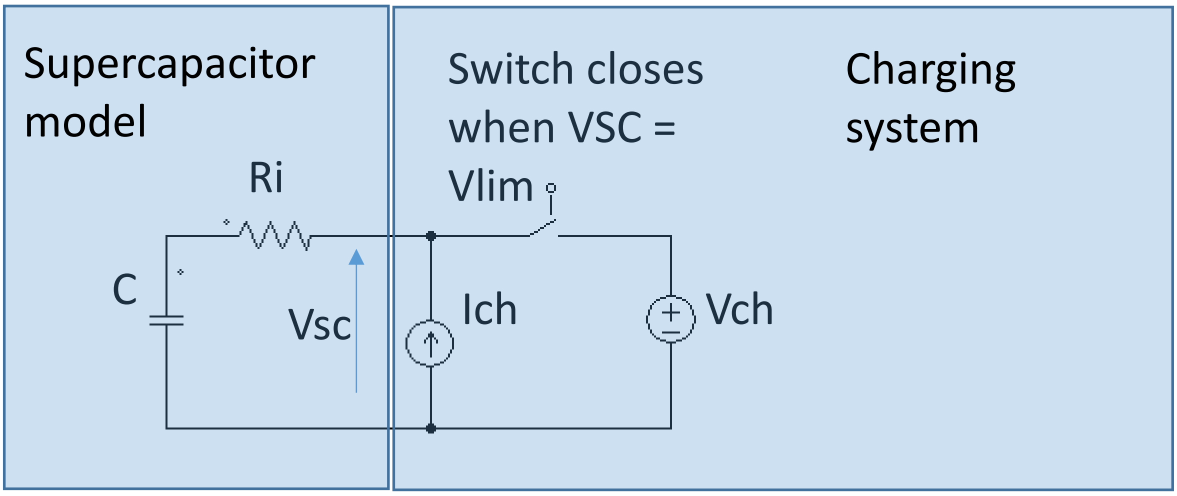

Figure 5.

Problem 4.7. The circuit of *Figure 6* is a passive differentiator, the voltage on v_{out} is $v_{out}(t) \approx dv_{in}/dt$. Which is the condition to be a "good" differentiator?

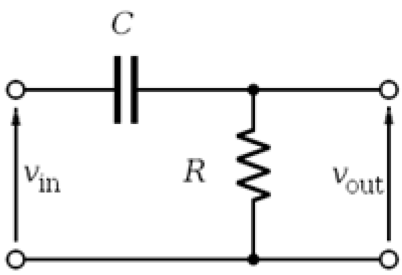

Figure 6.

Problem 4.8. The circuit of *Figure 7* is a passive integrator, the voltage on v_{out} is $v_{out}(t) \approx \int v_{in}\,dt$. Which is the condition to be a "good" integrator?

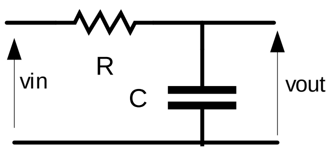

Figure 7.

BLOCK 5. MECHANICS

Problem 5.1. An oscillator is described by the following differential equation: $\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$. What is the condition for oscillator motion to be periodic with decreasing amplitude?

Problem 5.2. A solid cylinder of mass m and radius r is rolling on a perfectly rough surface with a constant velocity v. What is the kinetic energy of the cylinder?

Problem 5.3. A constant horizontal force F is acting on the center of the rolling cylinder from the previous problem. What is then the acceleration a of the cylinder?

Problem 5.4. Two identical solid balls with masses $m = 0.15$ kg are connected by a spring with constant $k = 0.726$ N/m. A third ball of the same mass $m = 0.15$ kg moves with velocity $v_0 = 1.2$ m/s and hits one of the two balls, the collision is absolutely elastic. What is the maximum amplitude x_{max} of spring oscillations after the collision?